Reference Sensor Selection for Improved Instantaneous Velocity Estimation Using Frequency Difference of Arrival Measurement

By

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ABSTRACT

In this paper, a reference sensor selection technique for a frequency difference of arrival (FDOA) based instantaneous velocity estimation algorithm was developed. This is to improve the instantaneous velocity estimation process of the algorithm. It involves condition number calculation of a matrix whose entries are the known instantaneous emitter position and the deployed sensor position. The sensor that resulted in the least condition number of the matrix is chosen as the reference for the velocity estimation process. Validation of the proposed technique was performed with the sensors in a triangular configuration. Monte Carlo simulation result comparison for the selected emitter position with the conventional approach shows a reduction in the velocity estimation error of at least 33%.

INTRODUCTION

The instantaneous velocity of a non-stationary emitting source can be determined from its electromagnetic emissions detected at spatially place sensors using a two-stage process (So, 2012). The first stage involves the estimation of velocity dependent signal parameters which is known as the frequency difference of arrival (FDOA) (So, 2012; Steffes & Rau, 2012). The next and final stage, which is the scope of this paper involves using the estimated velocity dependent signal parameter from the first stage with a lateration algorithm to estimate the instantaneous velocity of the emitter. With the knowledge of the instantaneous position of the emitter and the deployed sensors position, the instantaneous velocity of an emitter has a nonlinear relationship with the FDOA measurement. Several approaches have been developed to linearize this relationship (Ho, Lu, & Kovavisaruch, 2007; Ho & Xu, 2004; Kim, Kim, & Kim, 2015; Li, Guo, Yang, Jiang, & Pang, 2014; Wang, Cai, Li, & Ansari, 2016; Yang, An, & Xu, 2008; Yu, Huang, Gao, & Wu, 2012; Yu, Huang, Gao, & Yan, 2012; Zhu, Feng, Xie, & Zhou, 2016) which can be grouped into two namely: linear and non-linear approach (So, 2012). The non-linear approach involves the use of linearization techniques such as Taylor series method to linearize the relationship between the input variable (FDOA measurement) and output variable (emitter velocity) which is then followed by an iteration process (Yu, Huang, Gao, & Yan, 2012; Zhu et al., 2016). In the linear approach which is adopted in this paper involves the use of algebraic manipulation to obtain a linear relationship between the input and output variable (Chaitanya, Kumar, Rao, & Goswami, 2015). It does not suffer convergence issue due to lack of the iteration process but is said to produce high error in the velocity estimation process.

Several approaches have been proposed to improve velocity estimation of the FDOA-based lateration algorithm given perturb FDOA measurements (Jiandong, Xiaoyan, Pengyu, & Jiyong, 2008; Quo & Ho, 2011; Wang et al., 2016; Yang et al., 2008; Yu, Huang, Gao, & Wu, 2012). The use of techniques such as total least square (TLS) (Jiandong et al., 2008; Yang et al., 2008), approximates maximum likelihood algorithm (Yu, Huang, Gao, & Wu, 2012) and quadratic constrain solution approach (Quo & Ho, 2011) have been suggested. These techniques have been shown to greatly improve velocity estimation process of the FDOA-based lateration algorithm. The choice
of reference for the lateration algorithm has been shown to improve the lateration algorithm estimation process (Rene, Ortiz, Venegas, & Vidal, 2016; Xu, Lei, Cao, & Wei, 2014; Yaro, Sha’ameri, & Kamel, 2017). This has been implemented mostly in the time difference of arrival (TDOA) based lateration algorithm which is used in estimating the position of the emitting source (Sha’ameri, Shehu, & Asuti, 2015; Yaro, Musa, Sani, & Abdulaziz, 2016). For instance, a condition number based approach for reference sensor pair selection was presented in (Yaro et al., 2017) and has been shown to improve the emitter position estimation accuracy of the TDOA-based lateration algorithm. This paper proposed to improve the velocity estimation of the of the FDOA-based lateration algorithm using a reference sensor selection technique. A matrix whose entries are functions of the known instantaneous emitter position and the deployed sensors position is obtained. Prior to the estimation of the FDOA measurements, the condition number of the obtained matrix is obtained using each of the deployed sensors as reference. The sensor that resulted in the least condition number of the matrix is chosen as the reference for the estimation of the FDOA measurements to be used in the instantaneous velocity estimation of the emitter using the lateration algorithm.

The remainder of the paper is organized as follows: Section 2 gives a description of the variable reference instantaneous velocity estimation methodology which is followed by the description of the proposed FDOA estimation reference selection technique in Section 3. The simulation results and discussion are presented in Section 4 followed by the conclusion in Section 5.

VARIABLE REFERENCE INSTANTANEOUS VELOCITY ESTIMATION METHODOLOGY

Let \( x = (x_e, y_e) \) and \( v = (u_x, u_y) \) be the instantaneous position and velocity respectively of a non-stationary emitter. The frequency of arrival (FOA) also known as the Doppler frequency at the \( i \)-th sensor due its relative motion between the emitter and the sensor is mathematically obtained as:

\[
f_i(f) = \frac{f_c}{c} \times \left[ \frac{(x_e - x_i)u_x + (y_e - y_i)u_y}{R_i} \right]
\]

(1)

where \( f_c \) is the carrier frequency of the signal in Hz, \( c = 3 \times 10^8 \text{ m/s} \), \( s_i = (x_i, y_i) \) is the coordinate of the \( i \)-th sensor and \( R_i \) is the instantaneous Euclidean distance between the emitter and the \( i \)-th sensor which is mathematically expressed as:

\[
R_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}
\]

(2)

Given three stationary sensors, let the \( i \)-th sensor be chosen as reference for the FDOA estimation and the \( m \)-th and \( k \)-th sensor as the non-reference sensors with coordinates are \( s_m = (x_m, y_m) \) and \( s_k = (x_k, y_k) \) respectively. Two FDOA equations are obtained with the \( i \)-th sensor as reference which are as follows:

\[
f_{i,m} = f_i - f_m
\]

(3)

\[
f_{i,k} = f_i - f_k
\]

(4)

In practical application, the FDOA measurement given by Eq. (3) and Eq. (4) are obtained with error. This is due to the noise contained in the signal. Modelling this error as a Gaussian random variable with probability density function as \( N(0, \sigma) \) where \( \sigma \) is the FDOA estimation error standard deviation, the estimated FDOA measurements are:

\[
\hat{f}_{i,m} = f_{i,m} + N(0, \sigma)
\]

(5)

\[
\hat{f}_{i,k} = f_{i,k} + N(0, \sigma)
\]

(6)
Substituting Eq. (1) into Eq. (5) and Eq. (6), the FDOA equations can be expressed as functions of the instantaneous emitter position and velocity as follows:

\[ \hat{f}_{i,m} = \frac{f_c}{c} \left( \frac{(x_e - x_i)u_x}{R_i} + \frac{(y_e - y_i)u_y}{R_i} \right) - \frac{f_c}{c} \left( \frac{(x_e - x_m)u_x}{R_m} + \frac{(y_e - y_m)u_y}{R_m} \right) \]

(7)

\[ \hat{f}_{i,k} = \frac{f_c}{c} \left( \frac{(x_e - x_i)u_x}{R_i} + \frac{(y_e - y_i)u_y}{R_i} \right) - \frac{f_c}{c} \left( \frac{(x_e - x_k)u_x}{R_k} + \frac{(y_e - y_k)u_y}{R_k} \right) \]

(8)

It is assumed that the instantaneous emitter position is known thus, from Eq. (7) and Eq. (8), the unknown variable is the instantaneous velocity \((u_x, u_y)\). To solve for the known, there is a need to further simplify the Eq. (7) and Eq. (8) which respectively resulted into Eq. (9) and Eq. (10) as follows:

\[ u_x \times a_{11} + u_y \times a_{12} = \frac{\hat{f}_{i,m} \times c}{f_c} \]

(9)

\[ u_x \times a_{21} + u_y \times a_{22} = \frac{\hat{f}_{i,k} \times c}{f_c} \]

(10)

where the coefficients of Eq. (9) and Eq. (10) are as

\[ a_{11} = \frac{1}{R_i} (x - x_i) - \frac{1}{R_m} (x - x_m) \]

(11a)

\[ a_{12} = \frac{1}{R_i} (y - y_i) - \frac{1}{R_m} (y - y_m) \]

(11b)

\[ a_{21} = \frac{1}{R_i} (x - x_i) - \frac{1}{R_k} (x - x_k) \]

(11c)

\[ a_{22} = \frac{1}{R_i} (y - y_i) - \frac{1}{R_k} (y - y_k) \]

(11d)

The FDOA equations given in Eq. (9) and Eq. (10) can be written in matrix form as:

\[
\begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
  u_x \\
  u_y
\end{bmatrix}
= \begin{bmatrix}
  \frac{f_{12} \times c}{f_c} & -\frac{f_{13} \times c}{f_c} \\
  -\frac{f_{13} \times c}{f_c} & \frac{f_{12} \times c}{f_c}
\end{bmatrix}
\]

(12a)

\[ \mathbf{A}_i \mathbf{v} = \mathbf{b}_i \]

(12b)

Given the instantaneous emitter position \((x_e, y_e)\), estimated FDOA measurements \((\hat{f}_{i,m} \text{ and } \hat{f}_{i,k})\) and coordinates of the sensors, the estimated instantaneous velocity of the emitter can be obtained by finding the inverse matrix solution to Eq. (12) as follows:

\[
\begin{bmatrix}
  \hat{u}_x \\
  \hat{u}_y
\end{bmatrix}
= \begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}^{-1}
\begin{bmatrix}
  \frac{f_{12} \times c}{f_c} & -\frac{f_{13} \times c}{f_c} \\
  -\frac{f_{13} \times c}{f_c} & \frac{f_{12} \times c}{f_c}
\end{bmatrix}
\begin{bmatrix}
  \hat{f}_{i,m} \\
  \hat{f}_{i,k}
\end{bmatrix}
\]

(13a)

\[ \mathbf{v} = \left( \mathbf{A}_i \right)^{-1} \mathbf{b}_i \]

(13b)

The solution to Eq. (13) is the estimated instantaneous velocity with the \(i\)-th sensor as the reference for the FDOA estimation and the \(m\)-th and \(k\)-th as non-reference sensors.
PROPOSED FDOA ESTIMATION REFERENCE SELECTION TECHNIQUE

The choice of reference for estimating the FDOA measurement that is Eq. (5) and Eq. (6) influences the velocity estimation process of the algorithm presented in section 2. The velocity estimation process involves solving the least square (LS) equation in Eq. (12) given the instantaneous emitter position, estimated FDOA measurements and coordinates of the sensors. The effect of the error in the FDOA measurement on the solution of the LS in Eq. (12) is determined by the sensitivity of matrix $A_i$ (Golub & Van Loan, 2013). The sensitivity of any given matrix is defined by its condition number value. The condition number of a square matrix gives an indicates on how the error in the input variables is amplified to the solution obtained using the system (Golub & Van Loan, 2013). A higher condition number value indicates higher sensitivity to the input error which will result in a higher error in the velocity estimation process. The condition number of matrix $A_i$ in Eq. (12) denoted as $K(A_i)$ is obtained as (Ford, 2014; Golub & Van Loan, 2013):

$$K(A_i) = \|A_i\|_2 \times \|A_i^{-1}\|_2$$  \hspace{1cm} (14)

where $\|A_i\|_2$ and $\|A_i^{-1}\|_2$ respectively are the 2-norm of matrix $A_i$ and its inverse. The 2-norm of the matrix $A_{ij}$ and its inverse defined with respect to entries in matrix $A_i$ in Eq. (12) which are obtained from Eq. (11) are

$$\|A_i\|_2 = \sqrt{|a_{11}|^2 + |a_{12}|^2 + |a_{21}|^2 + |a_{22}|^2}$$ \hspace{1cm} (15a)

$$\|A_i^{-1}\|_2 = \sqrt{\frac{|a_{11}|^2 + |a_{12}|^2 + |a_{21}|^2 + |a_{22}|^2}{\det(A_i)}}$$ \hspace{1cm} (15b)

where $\det(A_{ij})$ is known as determinant of matrix $A_{ij}$ and is expressed mathematically as:

$$\det(A_{ij}) = (a_{11} \times a_{22}) - (a_{12} \times a_{21})$$ \hspace{1cm} (16)

Substituting Eq. (15) and Eq. (16) into Eq. (14), the condition number the matrix $A_i$ as function of it entries can be written as:

$$K(A_i) = \left(\frac{(|a_{11}|^2 + |a_{12}|^2 + |a_{21}|^2 + |a_{22}|^2)^2}{(a_{11} \times a_{22}) - (a_{12} \times a_{21})}\right)^{1/2}$$ \hspace{1cm} (17)

It can be seen that the entries of matrix $A_i$ in Eq. (12) is a function of the emitter instantaneous position and the sensor coordinates which are known. For an emitter at a fixed instantaneous position, different reference sensor for the FDOA estimation will produce different entries of matrix $A_i$. This will result in different condition number value in Eq. (17).

Considering the triangular sensor distribution as shown in Figure 1, Table 1 gives the condition number for matrix $A_i$ based on Eq. (17) for the difference reference sensor that is $i = 1, i = 2$ and $i = 3$ at some selected emitter instantaneous positions defined in cylindrical coordinate system.
Table 1: Matrix $A_i$ condition number for different sensor as reference. Blue shade indicates the sensor with the least $K(A_i)$

<table>
<thead>
<tr>
<th>Emitter position</th>
<th>$R_{range}$ (km)</th>
<th>Bearing (°)</th>
<th>for Reference sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>30, 4</td>
<td>2, 4</td>
</tr>
<tr>
<td>B</td>
<td>45</td>
<td>4, 2</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>60</td>
<td>5, 2</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>120</td>
<td>14, 21</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>135</td>
<td>11, 12</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td>150</td>
<td>10, 10</td>
<td>4</td>
</tr>
<tr>
<td>G</td>
<td>210</td>
<td>20, 11</td>
<td>24</td>
</tr>
<tr>
<td>H</td>
<td>225</td>
<td>26, 13</td>
<td>35</td>
</tr>
<tr>
<td>I</td>
<td>240</td>
<td>125, 136</td>
<td>261</td>
</tr>
</tbody>
</table>

From Table 1, the condition number differs for different emitter positions and reference sensor. At fixed emitter position, different reference sensor produces different condition numbers. At the instantaneous emitter position C, the condition numbers for reference sensor $i = 1$, $i = 2$ and $i = 3$ are 5, 2, 4 respectively. The reference sensor with the least condition number is $i = 2$ while $i = 5$ has the highest condition number value. At emitter position H, reference sensor $i = 1$ has the least condition number value of 125 while reference sensor $i = 3$ has the highest condition number value of 261. As earlier stated, the reference sensor with the least condition number is most suitable for the FDOA estimation as it will result in velocity estimates with the least error. This means that reference sensor $i = 1$ is the most suitable for estimating the instantaneous velocity of emitter at positions A, B, C, E and H, reference sensor $i = 3$ is the most suitable for estimating the instantaneous velocity at emitter positions D, E and F while reference sensor $i = 1$ is the most suitable for generating the FDOA measurements to estimate the instantaneous velocity of emitter at position I. Furthermore, based on the results in Table 1, the suitable reference sensor primarily depends on the emitter bearing.
measurement. Thus, summary of the procedure for selecting the suitable reference sensor for the FDOA estimation is described as follows:

i. Using the known instantaneous emitter position and sensor coordinates, obtain matrix $A_i$ using Eq. (11) and Eq. (12) for the difference reference sensor that is matrix $A_1$, matrix $A_2$ and matrix $A_3$ for $i = 1$, $i = 2$ and $i = 3$ respectively.

ii. Calculate the condition number for each of the matrix generated in (i) using Eq. (17).

iii. Chose the sensor with the least condition number value from (ii) as reference for generating the FDOA measurements in Eq. (5) and Eq. (6).

In the next Section, improvement in the instantaneous velocity estimation using the proposed reference sensor selection technique is presented.

**RESULT AND DISCUSSION**

In this section, the instantaneous velocity estimation accuracy of the algorithm presented in Section 2 using the proposed reference sensor selection technique in section 3 is compared with the conventional approach of using a fix reference sensor ($i = 1$) is presented. The comparison will be carried out for the selected emitter positions presented in Table 1 based on the sensor distribution in Figure 1. Instantaneous velocity root mean square error (RMSE) is used as the performance measure for the comparison and is mathematically expressed as follows:

$$V_{rmse} = \sqrt{\frac{\sum_{i=1}^{N} [(\hat{u}_{x,i} - u_x)^2 + (\hat{u}_{y,i} - u_y)^2]}{N}}$$

(18)

Where $(u_x, u_y)$ is the known instantaneous velocity and $(\hat{u}_{x,i}, \hat{u}_{y,i})$ is the estimated instantaneous velocity at the $i$-th Monte Carlo simulation realization while $N$ is the total number of Monte Carlo simulation realizations. Monte Carlo simulation results are obtained after $N = 500$ realizations and the instantaneous emitter velocity of $(u_x = 20 \text{ m/s}, u_y = 20 \text{ m/s})$ is considered for analysis.

By varying the FDOA error standard deviation from 0 Hz to 2 Hz, the instantaneous velocity RMSE obtained using the algorithm in section 2 with the proposed technique in Section 3 is compared with the conventional approach for the selected emitter position in Table 1. Figure 2 shows the velocity RMSE comparison between the two approaches. Result comparison between the two approaches shows that there is a linear relationship between the velocity RMSE and the FDOA error standard deviation. Summary of the velocity RMSE comparison at FDOA error standard deviation of 1 Hz for the selected emitter position can be seen in Table 2.

<table>
<thead>
<tr>
<th>Emitters position</th>
<th>Conventional approach</th>
<th>Proposed approach</th>
<th>Absolute difference</th>
<th>Percentage Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.28</td>
<td>0.19</td>
<td>0.09</td>
<td>47%</td>
</tr>
<tr>
<td>B</td>
<td>0.29</td>
<td>0.20</td>
<td>0.09</td>
<td>45%</td>
</tr>
<tr>
<td>C</td>
<td>0.30</td>
<td>0.21</td>
<td>0.09</td>
<td>43%</td>
</tr>
</tbody>
</table>

Table 2. Velocity RMSE comparison at FDOA error standard deviation of 1 Hz. Yellow shade indicates the approach with the least velocity RMSE.
From Table 2, irrespective of the emitter bearing, the velocity RMSE increases with increase in emitter range. For instance, at emitter position A with a range of 2 km, the velocity RMSE for the conventional approach and proposed approach are 0.28 m/s and 0.19 m/s respectively. At emitter position E with a range of 5 km, the velocity RMSE for the conventional and proposed approach increase to 1.43 m/s and 1.00 m/s respectively. Velocity RMSE comparison between the proposed and conventional approach for all emitter positions considered shows that there is an improvement in the velocity estimation process through a decrease in the estimation error. At emitter position B, the velocity RMSE using the conventional approach is 0.29 m/s which was decrease to about 0.20 m/s using the proposed approach. For emitter position with range of 2 km, the average reduction in velocity RMSE using the proposed approach in section 3 is about 0.09 m/s while for emitter positions with 5 km, the average reduction is about 0.48 km. As for emitter positions with range of 10 km, the velocity RMSE is very high using both approach but the average reduction in the error is about 2.84 m/s. The reference sensor chosen for the conventional approach corresponds to the same reference sensor obtained using the proposed technique at emitter position I. thus, the velocity RMSE for both approach is the same as presented in Table 2.
CONCLUSION

In this research, a technique based on condition number calculation to select the suitable reference sensor to improve the instantaneous velocity estimation of the FDOA-based system is proposed. Condition number calculation and velocity RMSE estimation comparison with the conventional technique was used to validate the proposed reference sensor selection technique. The condition number calculation analysis shows that the reference sensor suitable for the instantaneous velocity estimation using the FDOA-based algorithm has the least condition number value. Velocity RMSE based on Monte Carlo simulation shows that the proposed technique improves the estimation accuracy by reduction in the velocity RMSE of at least 33% based on the selected emitter positions with the sensors deployed in triangular configuration.

REFERENCES


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